## LOYOLA COLLEGE (AUTONOMOUS), CHENNAI - 600034

M.Sc. DEGREE EXAMINATION - MATHEMATICS

THIRD SEMESTER - APRIL 2023
PMT 3503 - OPERATIONS RESEARCH

Date: 06-05-2023
Time: 09:00 AM - 12:00 NOON

## Answer ALL the questions

I. (a) What do you understand by the term sensitivity analysis? Discuss the effect of variation of constant $b_{i}$.
(5 marks)
OR
(b) What is the meaning of upper bound and lower bound in branch and bound method?
(c) Solve the following integer programming problem using branch and bound technique:

Maximize $Z=10 x+20 y$
subject to $\quad 6 x+8 y \leq 48$
$x+3 y \leq 12$ where $x, y$ are non-negative integers.
OR
(d) Solve the following Linear Programming Problem.

Maximize $Z=3 x_{1}+5 x_{2}$

$$
\begin{equation*}
\text { subject to } \quad x_{1}+2 x_{2} \leq 9 \tag{15marks}
\end{equation*}
$$

$x_{2} \leq 4$ where $x_{1}, x_{2} \geq 0$
Discuss the effect of changing the availability of resources from $\left[\begin{array}{l}9 \\ 4\end{array}\right]$ to $\left[\begin{array}{c}12 \\ 7\end{array}\right]$ in the optimal solution. Also find out how far the new second resource can be increased.
II. (a)Explain Kendall's classification.

OR
(b) Explain any three selective inventory control techniques with examples from your life experience.
(5 marks)
(c) (i) What is the difference between under achievement and over achievement? How is goal programming useful to a company?
(ii) A company produces two products A and B. Each product must be processed through two departments. Department I has 72 hours of production capacity per week, and department II has 55 hours per week. Each unit of Product A requires 3 hours in department I and 4 hours in department II. Each unit of product B requires 4 hours in department I and 5 hours in department II. Management has set the following goals.

## Priority

Goal
$P_{1}$ : Minimize the underachievement of joint total production of 34 units.
$\mathrm{P}_{2}$ : Minimize the overachievement of producing 18 units of product A.
$P_{3}$ : Minimize the underachievement of producing 20 units of product $B$.
Formulate this problem as a GP problem and illustrate with graph.
marks)
(d) Perform ABC analysis for the items kept in inventory of a company and explain with graphical representation.
(15 marks)

| Item Name | Units | Unit cost in Rs. |
| :---: | :---: | :---: |
| 1 | 2000 | 11 |
| 2 | 300 | 15 |
| 3 | 800 | 8 |
| 4 | 4800 | 7 |
| 5 | 1200 | 16 |
| 6 | 18000 | 20 |
| 7 | 300 | 4 |
| 8 | 5000 | 9 |
| 9 | 500 | 12 |
| 10 | 4800 | 5 |

III. (a)What is replacement technique? Describe some replacement situations with examples.

OR
(b) Explain different types of failures in mechanism.
(c) A company wants to buy two types of machines. Machine A costs Rs. 80,000 . The maintenance costs are found to be as follows:

| YEAR | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Maintenance Cost | 1100 | 1300 | 1500 | 1900 | 2400 | 2900 | 3500 | 4300 |
| Resale value | 3100 | 1600 | 850 | 475 | 300 | 300 | 300 | 300 |

Machine B costs $30 \%$ more than Machine A. The maintenance costs are found to be as follows:

| YEAR | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Maintenance Cost | 1300 | 1600 | 1900 | 2500 | 3200 | 4100 | 5100 | 6200 |
| Resale value | 4100 | 2100 | 1100 | 600 | 400 | 400 | 400 | 400 |

The company plans to buy five machines with a combination of at least two machines of each type. Give the best combination to reduce the expense.
(15 marks)
OR
(d) (i) Explain different types of replacement policies. Which type of replacement policy do you follow at home? Why?
(ii) The cost of a machine is Rs. 7000 and its scrap value is only Rs. 300. The maintenance costs are found from experience are given below.

| Year | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Maintenance cost in Rs. | 100 | 250 | 400 | 600 | 900 | 1250 | 1600 | 2000 |

When should the machine be replaced?
(5+10
marks)
IV. (a) Discuss dynamic programming with suitable examples and compare with linear programming problem.

OR
(b) Define the terms stage, state, and recursive relation in dynamic programming problem. State Bellman's principle of optimality. marks)
(c) (i) Mention the characteristics of dynamic programming technique.
marks)
(ii) A group of students plan to travel from city 1 to city 10 so that the total cost becomes minimum. Travel cost from each city is given in the following table in hundreds of rupee. Find the least cost route from city 1 to city 10 using dynamic programming technique.


OR
(d) A medical company has six representatives to be assigned to three districts. How many of the six representatives should be assigned to each district in order to maximize the sale units?

| Representatives | District 1 | District 2 | District 3 |
| :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 0 |
| 1 | 25 | 20 | 33 |
| 2 | 42 | 38 | 43 |
| 3 | 55 | 54 | 47 |
| 4 | 63 | 65 | 50 |
| 5 | 69 | 73 | 52 |
| 6 | 74 | 80 | 53 |

(15 marks)
V. (a) Determine the values of $x, y, z$ that maximize or minimize the function $f(x, y)=-x^{2}-y^{2}-z^{2}+x y+10 x$.

OR
(b) State the necessary and sufficient Kuhn-Tucker conditions to solve quadratic programming problem.
(5 marks)
(c) Determine the maxima or minima of the function $\mathrm{f}=x^{2}+2 y^{2}+z^{2}+x y+z$ subject to the constraint $x+y+z=30$ at $x, y, z$ using Lagrangian Multiplier Methods.

OR
(d) Using Kuhn-Tucker conditions solve the non-linear programming problem:

Maximize $\mathrm{Z}=2 x_{1}^{2}-x_{2}$
subject to $x_{1}+x_{2}=7$
(15 marks)

$$
\begin{aligned}
& x_{1} \geq 1 \\
& x_{1}^{2}+x_{2}^{2} \leq 15 \text { where } x_{1}, x_{2} \geq 0 .
\end{aligned}
$$

